

2.12 Finite Laguerre Series

A. Purpose

This subroutine computes the value of a finite sum of Laguerre polynomials,

$$y = \sum_{j=0}^N a_j L_j(x)$$

for a specified summation limit, N , argument, x , and sequence of coefficients, a_j . The Laguerre polynomials are defined in [1].

B. Usage

B.1 Program Prototype, Single Precision

INTEGER N

REAL X, Y, A(0:m ≥ N)

Assign values to X, N, and A(0), A(1), ..., A(N).

CALL SLASUM (X, N, A, Y)

The sum will be stored in Y.

B.2 Argument Definitions

X [in] Argument of the polynomials.

N [in] Highest degree of polynomials in sum.

A() [in] The coefficients must be given in A(J), J = 0, ..., N.

Y [out] Computed value of the sum.

B.3 Modifications for Double Precision

For double precision usage, change the **REAL** statement to **DOUBLE PRECISION** and change the subroutine name from **SLASUM** to **DLASUM**.

C. Examples and Remarks

See **DRSLASUM** and **ODSLASUM** for an example of the usage of **SLASUM**. **DRSLASUM** evaluates the following identity, the coefficients of which were obtained from Table 22.10, page 799, of [1].

$$z = y - w = 0,$$

where

$$y = 7.2L_0(x) - 3.2L_1(x) + 108L_2(x) - 144L_3(x) \\ + 108L_4(x) - 43.2L_5(x) + 7.2L_6(x),$$

and

$$w = 0.01x^6.$$

D. Functional Description

The sum is evaluated by the following algorithm:

$$b_{N+2} = 0, \quad b_{N+1} = 0, \\ b_k = \frac{2k+1-x}{k+1}b_{k+1} - \frac{k+1}{k+2}b_{k+2} + a_k, \quad k = N, \dots, 0, \\ y = b_0.$$

For an error analysis applying to this algorithm see [2] and [3]. The first four Laguerre polynomials are

$$L_0(x) = 1, \quad L_1(x) = 1 - x, \\ L_2(x) = 1 - 2x + 0.5x^2, \\ L_3(x) = 1 - 3x + 1.5x^2 - (1/6)x^3.$$

For $k \geq 2$ the Laguerre polynomials satisfy the recurrence

$$kL_k(x) = (2k-1-x)L_{k-1}(x) - (k-1)L_{k-2}(x).$$

The Laguerre polynomials are orthogonal relative to integration with the weight function e^{-x} over the interval $[0, \infty)$, thus

$$\int_0^\infty e^{-x} L_i(x) L_j(x) dx = 0 \quad \text{if } i \neq j.$$

Laguerre polynomials are normally used only with an argument x satisfying $x \geq 0$.

References

1. Milton Abramowitz and Irene A. Stegun, **Handbook of Mathematical Functions**, *Applied Mathematics Series 55*, National Bureau of Standards (1966) Chapter 22, 771–802.
2. E. W. Ng, *Direct summation of series involving higher transcendental functions*, **J. Comp. Phys.** **3**, 2 (Oct. 1968) 334–338.
3. E. W. Ng, *Recursive algorithm for the computation of hypergeometric series*, **SIAM J. on Math. Anal.** **2** (1971) 31–36.

E. Error procedures and Restrictions

The subroutine will return Y = 0 if N < 0. It is recommended that X satisfy X ≥ 0.

F. Supporting Information

The source language is ANSI Fortran 77.

Entry **Required Files**

DLASUM **DLASUM**

SLASUM **SLASUM**

Based on a 1974 program by E.W. Ng, JPL. Present version by C.L. Lawson and S. Y. Chiu, JPL, 1983.

DRSLASUM

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c      DRSLASUM
c>> 1994-10-19 DRSLASUM Krogh  Changes to use M77CON
c>> 1994-07-14 DRSLASUM CLL
c>> 1992-05-07 CLL
c>> 1992-04-28 DRSLASUM Replaced '1' in format.
c>> 1987-12-09 DRSLASUM Lawson Initial Code.
c—S replaces "?": DR?LASUM, ?LASUM
c      Demonstration program for evaluation of a Laguerre series.
c
      integer j
      real      x,a(0:6),y,w,dif,reldif,pn(6)
      data a / 7.2e0, -43.2e0, 108.0e0, -144.0e0,
*            108.0e0, -43.2e0, 7.2e0 /
      data pn / 0.1e0, 0.3e0, 1.0e0, 3.0e0, 10.0e0, 30.0e0 /
c
      print '(1x,3x,a1,12x,a1,14x,a3,9x,a6/)', 'x','y','dif','reldif'
      do 10 j = 1,6
        x = pn(j)
        call slasum (x, 6, a, y)
        w = 0.01e0 * (x**6)
        dif = y - w
        reldif = dif / w
        print '(1x,f6.2,3x,g15.8,2(3x,g10.3))',x,y,dif,reldif
10 continue
      end

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ODSLASUM

x	y	dif	reldif
0.10	-0.11444092E-04	-0.115E-04	-0.115E+04
0.30	0.95367432E-05	0.225E-05	0.308
1.00	0.10004997E-01	0.500E-05	0.500E-03
3.00	7.2899961	-0.381E-05	-0.523E-06
10.00	10000.000	0.00	0.00
30.00	7290000.0	0.00	0.00